

Part 4 of 5:

A New MMP Method

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Summary. In this paper, we explain how proportional representation by the single transferable vote can be incorporated into a mixed member proportional representation scheme.

Keywords and Phrases: mixed member proportional representation, proportional rankings, single transferable vote

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This paper is the fourth part of a series of papers that can be downloaded here:

<http://m-schulze.webhop.net/schulze1.pdf>
<http://m-schulze.webhop.net/schulze2.pdf>
<http://m-schulze.webhop.net/schulze3.zip>
<http://m-schulze.webhop.net/schulze4.pdf>
<http://m-schulze.webhop.net/schulze5.pdf>

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1. Introduction

Today, many countries use *proportional representation by party lists* (PRPL). This election method has two serious problems. First, it gives too much power to the party machines that control the nomination processes. Second, it allows proportionality only according to one criterion: party affiliations.

A possible way to circumvent the shortcomings of PRPL is through *proportional representation by the single transferable vote* (STV). Here, each voter gets a complete list of all candidates and ranks these candidates in order of preference. The ballots are then counted in such a manner that the following property (*Droop proportionality*) is satisfied:

Suppose N is the number of voters and M is the number of seats. Then: If $N_1 > 0$ voters strictly prefer every candidate of a given set of C_1 candidates to every other candidate, then at least $\min \{ \lceil (M+1) \cdot N_1/N \rceil - 1; C_1 \}$ candidates of this set must be elected.

To cope with the large number of candidates who are typically running for a parliament, the electorate is divided into districts of e.g. 5 seats each and the same candidate must not run in more than one district. Droop proportionality is then satisfied only on the district level, so that it can happen that a party with about 15% of the votes does not win a single seat.

Therefore, people who promote STV methods in countries that use PRPL are frequently confronted with the defamation that they were dishonest and that their real aim was to increase the threshold for small parties to gain parliamentary representation from a *legal* threshold of typically about 5% in countries that are using PRPL to a *natural* threshold of typically about 15% in countries that are using STV. Because of this reason, it might be a useful strategy to include into the STV proposal a compensation of party proportionality on the national level.

One way to achieve this compensation of party proportionality is through *mixed member proportional representation* (MMP). Here, each voter gets two ballots: a *district ballot* and a *party ballot*. With the district ballots, the voters elect their district representatives. With the party ballots, the voters indicate their party support. When, on the national level, the elected district representatives do not reflect party proportionality (as indicated by the party ballots) in an appropriate manner, then *additional representatives* are added to the parliament to compensate party proportionality (so that the total size of the parliament increases). Usually, these additional representatives are chosen from candidate lists that have been submitted by the parties in advance of the elections. The main problem of this way to choose these additional representatives is that, again, it gives too much power to the party machines that control the nomination processes.

One way to avoid this problem of the common way to choose these additional representatives is to use MMP with the "*best loser*" method. Here, when a party gets additional representatives, then these additional representatives are those candidates who performed best in their respective districts without being elected. Advantage of the "best loser" method is that, with their district votes, the voters do not only elect their district representatives, they also decide (in those cases where a party gets additional representatives) who these additional representatives are.

However, to be able to combine STV with the "best loser" method, we need a heuristic that tells us how well the candidates performed in their respective districts. Here, we propose that, in each district, a *proportional ranking* of all candidates should be calculated. A proportional ranking is a complete ranking (*linear ordering*) of all candidates such that, for every Z , the first Z candidates reflect the voters as proportionally as possible. We propose that the *Schulze proportional ranking* should be used, as described in section 6 of the second paper of this series of papers. When a party wins additional representatives, then those candidates of this party are elected who did not win a district seat, but who are ranked highest in the proportional rankings of their respective districts.

In section 2 of this paper, we introduce the details of the district vote. We explain the creation of the districts (section 2.1), the district ballot (section 2.2), and the determination of the district vote winners (section 2.3). In section 3, we explain the details of the party vote. The party ballot is illustrated in section 3.1. In sections 3.2 and 3.3, we show how to determine how many additional representatives a given party gets in a given district. In section 3.4, these additional representatives are chosen. In section 4, vacant seats are filled.

Where concrete numbers are needed, we use the elections to the *Berlin House of Representatives (Abgeordnetenhaus von Berlin)* to illustrate the proposed method. Currently, the electoral law says that the House consists of at least 130 members. 78 members (= 60%) are elected by *first-past-the-post* (FPP) in single winner districts (*single member plurality*, SMP); at least additional 52 members are elected by closed party lists to compensate party proportionality. However, as FPP does not lead to proportional results, usually significantly more than 52 additional members are needed to compensate party proportionality, so that the House usually has a size of about 150 members. We recommend that, in future, 115 of the 130 members (about 90%) should be elected by STV in districts of 8 to 22 seats. This is possible without creating too many additional members, because STV already leads to very proportional results.

2. The District Vote

2.1. The Districts

Berlin is currently divided into 12 boroughs (sing.: *Bezirk*, plur.: *Bezirke*).

	borough	eligible voters (on 17 Sep. 2006)
1	Mitte	190,550
2	Friedrichshain-Kreuzberg	165,331
3	Pankow	274,380
4	Charlottenburg-Wilmersdorf	216,374
5	Spandau	160,411
6	Steglitz-Zehlendorf	213,787
7	Tempelhof-Schöneberg	231,249
8	Neukölln	193,014
9	Treptow-Köpenick	193,936
10	Marzahn-Hellersdorf	201,209
11	Lichtenberg	201,096
12	Reinickendorf	184,143
	total:	2,425,480

Table 2.1.1: The 12 Berlin boroughs



Table 2.1.2: The 12 Berlin boroughs

We recommend that the districts for the elections to the Berlin House of Representatives should be the 12 Berlin boroughs. When the Hill-Huntington method is being used to allocate the 115 district seats to the 12 districts, then we get two 8-seat districts (Friedrichshain-Kreuzberg, Spandau), five 9-seat districts (Mitte, Neukölln, Treptow-Köpenick, Lichtenberg, Reinickendorf), three 10-seat districts (Charlottenburg-Wilmersdorf, Steglitz-Zehlendorf, Marzahn-Hellersdorf), one 11-seat district (Tempelhof-Schöneberg), and one 13-seat district (Pankow). See table 2.1.3.

Alternatively, Berlin could be divided into 6 districts: Central (Mitte and Friedrichshain-Kreuzberg), North (Pankow and Reinickendorf), West (Charlottenburg-Wilmersdorf and Spandau), South-West (Steglitz-Zehlendorf and Tempelhof-Schöneberg), South-East (Neukölln and Treptow-Köpenick), and East (Marzahn-Hellersdorf and Lichtenberg). With the Hill-Huntington method, we would then get one 17-seat district (Central), two 18-seat districts (West, South-East), one 19-seat district (East), one 21-seat district (South-West), and one 22-seat district (North).

district	eligible voters	number of eligible voters divided by ...													
		... $\sqrt{(1\cdot 2)}$... $\sqrt{(2\cdot 3)}$... $\sqrt{(3\cdot 4)}$... $\sqrt{(4\cdot 5)}$... $\sqrt{(5\cdot 6)}$... $\sqrt{(6\cdot 7)}$... $\sqrt{(7\cdot 8)}$... $\sqrt{(8\cdot 9)}$... $\sqrt{(9\cdot 10)}$... $\sqrt{(10\cdot 11)}$... $\sqrt{(11\cdot 12)}$... $\sqrt{(12\cdot 13)}$... $\sqrt{(13\cdot 14)}$	
Mitte	190,550	134,739 (21. seat)	77,791 (34. seat)	55,007 (46. seat)	42,608 (58. seat)	34,789 (72. seat)	29,402 (83. seat)	25,463 (97. seat)	22,456 (109. seat)	20,085	18,168	16,585	15,256	14,124	
Friedrichshain-Kreuzberg	165,331	116,906 (23. seat)	67,496 (36. seat)	47,726 (52. seat)	36,969 (64. seat)	30,185 (80. seat)	25,511 (95. seat)	22,093 (110. seat)	19,484	17,427	15,763	14,390	13,237	12,255	
Pankow	274,380	194,015 (13. seat)	112,015 (25. seat)	79,206 (31. seat)	61,353 (41. seat)	50,094 (49. seat)	42,337 (59. seat)	36,665 (67. seat)	32,335 (76. seat)	28,922 (85. seat)	26,161 (92. seat)	23,881 (102. seat)	21,967 (112. seat)	20,338	
Charlottenburg-Wilmersdorf	216,374	152,999 (15. seat)	88,334 (27. seat)	62,461 (39. seat)	48,382 (50. seat)	39,504 (62. seat)	33,387 (74. seat)	28,914 (86. seat)	25,499 (96. seat)	22,807 (106. seat)	20,630	18,832	17,323	16,038	
Spandau	160,411	113,427 (24. seat)	65,487 (38. seat)	46,306 (53. seat)	35,868 (68. seat)	29,286 (84. seat)	24,751 (99. seat)	21,435 (114. seat)	18,904	16,908	15,294	13,961	12,843	11,890	
Steglitz-Zehlendorf	213,787	151,170 (16. seat)	87,278 (28. seat)	61,714 (40. seat)	47,804 (51. seat)	39,031 (63. seat)	32,988 (75. seat)	28,568 (87. seat)	25,195 (98. seat)	22,535 (108. seat)	20,383	18,607	17,116	15,846	
Tempelhof-Schöneberg	231,249	163,517 (14. seat)	94,407 (26. seat)	66,755 (37. seat)	51,708 (48. seat)	42,220 (60. seat)	35,682 (69. seat)	30,901 (79. seat)	27,252 (89. seat)	24,375 (101. seat)	22,048 (111. seat)	20,127	18,514	17,141	
Neukölln	193,014	136,481 (20. seat)	78,797 (33. seat)	55,718 (45. seat)	43,159 (57. seat)	35,239 (71. seat)	29,782 (82. seat)	25,792 (94. seat)	22,746 (107. seat)	20,345	18,403	16,799	15,453	14,307	
Treptow-Köpenick	193,936	137,133 (19. seat)	79,174 (32. seat)	55,984 (44. seat)	43,365 (56. seat)	35,407 (70. seat)	29,924 (81. seat)	25,915 (93. seat)	22,855 (105. seat)	20,442	18,491	16,879	15,527	14,375	
Marzahn-Hellersdorf	201,209	142,276 (17. seat)	82,143 (29. seat)	58,084 (42. seat)	44,991 (54. seat)	36,735 (65. seat)	31,047 (77. seat)	26,887 (90. seat)	23,712 (103. seat)	21,209 (115. seat)	19,184	17,512	16,109	14,914	
Lichtenberg	201,096	142,196 (18. seat)	82,097 (30. seat)	58,051 (43. seat)	44,966 (55. seat)	36,714 (66. seat)	31,029 (78. seat)	26,872 (91. seat)	23,699 (104. seat)	21,197	19,173	17,503	16,100	14,906	
Reinickendorf	184,143	130,208 (22. seat)	75,176 (35. seat)	53,157 (47. seat)	41,175 (61. seat)	33,619 (73. seat)	28,413 (88. seat)	24,607 (100. seat)	21,701 (113. seat)	19,410	17,557	16,027	14,743	13,649	

Table 2.1.3: Allocation of the 115 district seats to the 12 districts according to the Hill-Huntington method

When the Hill-Huntington method is being used, then, at the first stage, each district gets one seat. At the second stage, the numbers of eligible voters of each district are divided by $\sqrt{(1\cdot 2)}$, $\sqrt{(2\cdot 3)}$, $\sqrt{(3\cdot 4)}$, $\sqrt{(4\cdot 5)}$, $\sqrt{(5\cdot 6)}$, ... and the remaining seats go to the largest quotients.

2.2. The District Ballot

The same candidate cannot run in more than one district. The same candidate cannot run simultaneously as an independent candidate and as a party candidate. The same candidate cannot run for more than one party simultaneously.

On the district ballot, the candidates are sorted according to their party affiliations. Candidates with the same party affiliation are sorted in an order determined by this party.

The individual voter ranks the candidates in order of preference. The individual voter may ...

... give the same preference to more than one candidate.

... keep candidates unranked. When a given voter does not rank all candidates, then this means (1) that this voter strictly prefers all ranked candidates to all not ranked candidates and (2) that this voter is indifferent between all not ranked candidates.

... skip preferences. However, skipping some preferences does not have any impact on the result of the elections, since the result of the elections depends only on the order in which the individual voters ranks the candidates and not on the absolute preferences of the individual voters.

... give preferences to parties. When a given voter gives a preference to a party, then this means that each candidate of this party gets this preference unless this voter explicitly gives a different preference to this candidate.

Table 2.2.1 shows how a cast district ballot could look like. Table 2.2.2 illustrates how this district ballot would be interpreted.

Elections to the **Berlin House of Representatives**
on 17. September 2006

District Ballot

for district Friedrichshain-Kreuzberg

*please rank the candidates
in order of preference*



01: Social Democratic Party of Germany (SPD)	14
01.001: Junge-Reyer , Ingeborg	23
01.002: Zackenfels , Stefan	7
01.003: Kitschun , Susanne	
01.004: Eggert , Björn	
01.005: Bayram , Canan	
01.006: Fischer , Silke	
01.007: Heinemann , Sven	23
01.008: Miethke , Petra	23
01.009: Kayhan , Sevgi	6
01.010: Erdem , Hediye	8
01.011: Klebba , Sigrid	13
01.012: Postler , Lorenz	
01.013: Hehmke , Andy	
01.014: Dr. Beckers , Peter	19
01.015: Lorenz , Dorit	24
01.016: Borchard , Andreas	
02: Christian Democratic Union of Germany (CDU)	
02.001: Wansner , Kurt	
02.002: Bleiler , Rainer	
02.003: Ruhland , Thomas	
02.004: Samuray , Sedat	16
02.005: Stry , Ernst-Uwe	
02.006: Rösner , Helga	14
02.007: Glatzel , Edgar	20
02.008: Schill , Michael	
02.009: Müller , Götz	
02.010: Freitag , Jens-Matthias	
02.011: Husein , Timur	
02.012: Wöhrn , Marina	
02.013: Taşkıran , Ertan	
02.014: Przewieslik , Wolfgang	
02.015: Konschak , Benjamin	
02.016: Bohl , Daniel-Stephan	
03: Left Party	15
03.001: Michels , Martina	11
03.002: Wolf , Udo	14
03.003: Matuschek , Jutta	23
03.004: Zillich , Steffen	16
03.005: İzgin , Figen	5
03.006: Günther , Andreas	
03.007: Vordenbäumen , Vera	13
03.008: Krüger , Wolfgang	
03.009: Reinauer , Cornelia	
03.010: Bauer , Kerstin	
03.011: Mildner-Spindler , Knut	
03.012: Richter , Claudia	6
03.013: Schüssler , Lothar	16
03.014: Thimm , Helga	10
03.015: Pempel , Joachim	21
03.016: Sommer-Wetter , Regine	
04: Alliance '90 / The Greens (B'90G)	15
04.001: Ratzmann , Volker	16
04.002: Mutlu , Özcan	2
04.003: Dr. Klotz , Sibyll-Anka	12
04.004: Lux , Benedikt	
04.005: Herrmann , Clara	17
04.006: Stephan , André	9
04.007: Pohner , Wolfgang	
04.008: Dr. Altug , Mehmet	
04.009: Burkert-Eulitz , Marianne	13
04.010: Kosche , Heidi	23
04.011: Behrendt , Dirk	1
04.012: Hauser-Jabs , Christine	12
04.013: Schulz , Franz	4
04.014: Kapek , Antje	3
04.015: Wesener , Daniel	22
04.016: Çetinkaya , İstikbal	13
05: Free Democratic Party of Germany (FDP)	20
05.001: Peters , Frank	21
05.002: Dr. Hansen , Nikoline	
05.003: Eydner , John	
05.004: Hohl , Heinrich	
05.005: Salonek , Gumbert-Olaf	23
05.006: Diener , Thomas	
05.007: Schaefer , Martina	
05.008: Wolf , Tobias	
05.009: Dr. Stolz , Peter	
05.010: Lauf , Sebastian	
05.011: Paun , Christopher	21
05.012: Joecken , Ilka	
06: The Republicans (REP)	
06.001: Dr. Clemens , Björn	
06.002: Kuhn , Daniel	
06.003: Hinze , Harald Björn Gunnar	
06.004: Nestmann , Günther	
07: Ecological Democratic Party (ödp)	18
07.001: Machel-Ebeling , Johannes	
08: Civil Rights Movement Solidarity (BüSo)	22
08.001: Hinz , Björn	
09: Humane Economy Party	18
09.001: Dr. Heinrichs , Johannes	
10.001: Eisner , Udo (independent)	21
11.001: Stiewe , Hauke (independent)	20

Table 2.2.1: District ballot

Elections to the **Berlin House of Representatives**
on 17. September 2006

District Ballot

for district Friedrichshain-Kreuzberg

*please rank the candidates
in order of preference*



01: Social Democratic Party of Germany (SPD)	
01.001: Junge-Reyer , Ingeborg	23
01.002: Zackenfels , Stefan	7
01.003: Kitschun , Susanne	14
01.004: Eggert , Björn	14
01.005: Bayram , Canan	14
01.006: Fischer , Silke	14
01.007: Heinemann , Sven	23
01.008: Miethke , Petra	23
01.009: Kayhan , Sevgi	6
01.010: Erdem , Hediye	8
01.011: Klebba , Sigrid	13
01.012: Postler , Lorenz	14
01.013: Hehmke , Andy	14
01.014: Dr. Beckers , Peter	19
01.015: Lorenz , Dorit	24
01.016: Borchard , Andreas	14
02: Christian Democratic Union of Germany (CDU)	
02.001: Wansner , Kurt	25
02.002: Bleiler , Rainer	25
02.003: Ruhland , Thomas	25
02.004: Samuray , Sedat	16
02.005: Stry , Ernst-Uwe	25
02.006: Rösner , Helga	14
02.007: Glatzel , Edgar	20
02.008: Schill , Michael	25
02.009: Müller , Götz	25
02.010: Freitag , Jens-Matthias	25
02.011: Husein , Timur	25
02.012: Wöhrn , Marina	25
02.013: Taşkıran , Ertan	25
02.014: Przewieslik , Wolfgang	25
02.015: Konschak , Benjamin	25
02.016: Bohl , Daniel-Stephan	25
03: Left Party	
03.001: Michels , Martina	11
03.002: Wolf , Udo	14
03.003: Matuschek , Jutta	23
03.004: Zillich , Steffen	16
03.005: İzgin , Figen	5
03.006: Günther , Andreas	15
03.007: Vordenbäumen , Vera	13
03.008: Krüger , Wolfgang	15
03.009: Reinauer , Cornelia	15
03.010: Bauer , Kerstin	15
03.011: Mildner-Spindler , Knut	15
03.012: Richter , Claudia	6
03.013: Schüssler , Lothar	16
03.014: Thimm , Helga	10
03.015: Pempel , Joachim	21
03.016: Sommer-Wetter , Regine	15
04: Alliance '90 / The Greens (B'90G)	
04.001: Ratzmann , Volker	16
04.002: Mutlu , Özcan	2
04.003: Dr. Klotz , Sibyll-Anka	12
04.004: Lux , Benedikt	15
04.005: Herrmann , Clara	17
04.006: Stephan , André	9
04.007: Pohner , Wolfgang	15
04.008: Dr. Altug , Mehmet	15
04.009: Burkert-Eulitz , Marianne	13
04.010: Kosche , Heidi	23
04.011: Behrendt , Dirk	1
04.012: Hauser-Jabs , Christine	12
04.013: Schulz , Franz	4
04.014: Kapek , Antje	3
04.015: Wesener , Daniel	22
04.016: Çetinkaya , İstikbal	13
05: Free Democratic Party of Germany (FDP)	
05.001: Peters , Frank	21
05.002: Dr. Hansen , Nikoline	20
05.003: Eydner , John	20
05.004: Hohl , Heinrich	20
05.005: Salonek , Gumbert-Olaf	23
05.006: Diener , Thomas	20
05.007: Schaefer , Martina	20
05.008: Wolf , Tobias	20
05.009: Dr. Stolz , Peter	20
05.010: Lauf , Sebastian	20
05.011: Paun , Christopher	21
05.012: Joecken , Ilka	20
06: The Republicans (REP)	
06.001: Dr. Clemens , Björn	25
06.002: Kuhn , Daniel	25
06.003: Hinze , Harald Björn Gunnar	25
06.004: Nestmann , Günther	25
07: Ecological Democratic Party (ödp)	
07.001: Machel-Ebeling , Johannes	18
08: Civil Rights Movement Solidarity (BüSo)	
08.001: Hinz , Björn	22
09: Humane Economy Party	
09.001: Dr. Heinrichs , Johannes	18
10.001: Eisner , Udo (independent)	21
11.001: Stiewe , Hauke (independent)	20

Table 2.2.2: District ballot

2.3. The District Vote Winners

In each district, a proportional ranking of all candidates is calculated. For the district Friedrichshain-Kreuzberg, this proportional ranking could look as follows:

- | | |
|-------------------------------------------|---------------------------|
| 1. Mutlu (B'90G, candidate 04.002) | → elected (district vote) |
| 2. Fischer (SPD, candidate 01.006) | → elected (district vote) |
| 3. Reinauer (Left, candidate 03.009) | → elected (district vote) |
| 4. Junge-Reyer (SPD, candidate 01.001) | → elected (district vote) |
| 5. Ratzmann (B'90G, candidate 04.001) | → elected (district vote) |
| 6. İzgin (Left, candidate 03.005) | → elected (district vote) |
| 7. Klotz (B'90G, candidate 04.003) | → elected (district vote) |
| 8. Samuray (CDU, candidate 02.004) | → elected (district vote) |
| 9. Eggert (SPD, candidate 01.004) | |
| 10. Heinemann (SPD, candidate 01.007) | |
| 11. Behrendt (B'90G, candidate 04.011) | |
| 12. Wolf (Left, candidate 03.002) | |
| 13. Altug (B'90G, candidate 04.008) | |
| 14. Bayram (SPD, candidate 01.005) | |
| 15. Kosche (B'90G, candidate 04.010) | |
| 16. Michels (Left, candidate 03.001) | |
| 17. Bleiler (CDU, candidate 02.002) | |
| 18. Herrmann (B'90G, candidate 04.005) | |
| 19. Zackenfels (SPD, candidate 01.002) | |
| 20. Vordenbäumen (Left, candidate 03.007) | |
| 21. etc. | |

The idea is: If only the SPD supporters had participated, then this proportional ranking would have been Fischer, Junge-Reyer, Eggert, Heinemann, Bayram, Zackenfels, etc.. If only the B'90G supporters had participated, then this proportional ranking would have been Mutlu, Ratzmann, Klotz, Behrendt, Altug, Kosche, Herrmann, etc.. If only the Left Party supporters had participated, then this proportional ranking would have been Reinauer, İzgin, Wolf, Michels, Vordenbäumen, etc.. If only the CDU supporters had participated, then this proportional ranking would have been Samuray, Bleiler, etc..

As Friedrichshain-Kreuzberg is an 8-seat district, the first 8 candidates of this proportional ranking are elected.

3. The Party Vote

3.1. The Party Ballot

On the party ballot of a given district, all those parties are listed that have nominated district candidates. The individual voter can vote for one and only one party. Table 3.1.1 shows how a cast ballot for district Friedrichshain-Kreuzberg could look like.

Elections to the **Berlin House of Representatives**
on 17. September 2006

Party Ballot

for district Friedrichshain-Kreuzberg

*please vote for one
and only one party*

01: Social Democratic Party of Germany (SPD)	<input type="radio"/>
02: Christian Democratic Union of Germany (CDU)	<input type="radio"/>
03: Left Party	<input type="radio"/>
04: Alliance '90 / The Greens (B'90G)	<input checked="" type="radio"/>
05: Free Democratic Party of Germany (FDP)	<input type="radio"/>
06: The Republicans (REP)	<input type="radio"/>
07: Ecological Democratic Party (ödp)	<input type="radio"/>
08: Civil Rights Movement Solidarity (BüSo)	<input type="radio"/>
09: Humane Economy Party	<input type="radio"/>

Table 3.1.1: Party ballot

3.2. Allocation of Seats to Parties

Suppose X is the number of district seats won by independent candidates.

Suppose D_i is the number of district seats won by party i .

Suppose P_i is the number of party votes for party i .

Suppose S_i is the number of seats that will have been allocated to party i during the allocation process.

We recommend that the rules to allocate the seats to the parties should have the following properties:

- The seats are allocated according to the Sainte-Laguë method with 0.75 as first divisor. That means: The numbers of party votes for each party are divided by 0.75, 1.5, 2.5, 3.5, 4.5, 5.5, ... and the seats go to the largest quotients.
- When a given party wins more district seats than it deserves seats according to its number of party votes, then this party keeps all these district seats, so that the total size of the Berlin House of Representatives increases (*overhang seats*). The other parties then also get additional seats to preserve party proportionality, so that the total size of the House increases further (*compensation seats*). However, the total size of the House should not be larger than absolutely necessary to preserve party proportionality, because every additional seat compromises the idea that the House should be elected by proportional representation by the single transferable vote.
- However, to guarantee that the size of the House does not vary too much from one election to the other election, the minimum size is set to 131 members and the maximum size is set to 179 members. Therefore, we get $131 \leq X + \sum_j S_j \leq 179$.
- The constitution of Berlin says that a party has qualified if and only if it has received at least 5% of the valid party votes or has won at least one district seat. If a party has not qualified, then it must not get any seats (*threshold clause*).
- Suppose $P_{\text{qual}} := \sum_j (P_j \mid \text{party } j \text{ has qualified})$.

In Germany, it is tradition that, if party i has qualified and has received more than half of the votes of the qualified parties, then party i must also get more than half of the seats that are allocated to qualified parties (*majority clause*). Therefore, we get: If party i has qualified and $P_i > P_{\text{qual}} / 2$, then $S_i > (\sum_j S_j) / 2$.

- The total number of seats $X + \sum_j S_j$ should be odd (*stalemate clause*).

Therefore, we propose the following method to allocate the seats to the parties:

s_i is the number of seats already allocated to party i .

We will concentrate on the qualified parties. Parties, that have not qualified, will be ignored in the following allocation procedure. If no party has qualified, then no seats will be allocated on the city-wide level.

Stage 1:

For each party i , we start with $s_i := D_i$.

Stage 2:

$r_i := P_i / (D_i - 0.5)$ for each party i with $D_i > 1$.

$r_i := P_i / 0.75$ for each party i with $D_i = 1$.

$r_i := \infty$ for each party i with $D_i = 0$.

$Y := \min_i r_i$.

[The idea is: As soon as the next quotient T , that will be rewarded with a seat, is equal to or strictly smaller than Y , party proportionality has been achieved, so that the allocation procedure can stop. Adding more seats would not improve proportionality according to party affiliations any further, but would worsen proportionality according to whichever other criteria the voters considered important when choosing the district winners by STV.]

Stage 3:

Repeat (until at least one of the termination conditions is satisfied):

$$t_i := P_i / (s_i + 0.5) \text{ for each party } i \text{ with } s_i > 0.$$

$$t_i := P_i / 0.75 \text{ for each party } i \text{ with } s_i = 0.$$

$$T := \max_i t_i.$$

$$U := \max_i \{ D_i \mid t_i = T \}.$$

If at least one of the following two conditions is satisfied, then the allocation procedure terminates.

Condition #1:

The following four statements are satisfied:

(a) $X + \sum_j s_j \geq 131$.

(b) $X + \sum_j s_j$ is odd.

(c) $T \leq Y$.

(d) There is no party i with
 $P_i > P_{\text{qual}} / 2$ and $s_i \leq (\sum_j s_j) / 2$.

Condition #2:

$$X + \sum_j s_j = 179.$$

If none of these two conditions is satisfied, then we proceed as follows:

If there is a party i with $P_i > P_{\text{qual}} / 2$ and $s_i \leq (1 + \sum_j s_j) / 2$, then the next seat goes to party i .
 Otherwise:

The next seat goes to party i with $t_i = T$ and $D_i = U$. If there is more than one party with $t_i = T$ and $D_i = U$, then we decide randomly which party with $t_i = T$ and $D_i = U$ gets the next seat.

3.3. Allocation of the Party Seats to this Party's District Organizations

As candidates run on the district level and not on a city-wide level, we now have to allocate the seats of a party to this party's district organizations. We recommend that this allocation procedure should have the following properties:

- The seats are allocated according to the Sainte-Laguë method with 0.75 as first divisor.
- Each district organization must get at least as many seats as it has won district seats.

Therefore, we propose the following method to allocate the seats of party i to this party's district organizations:

Suppose D_i^j is the number of district seats won by party i in district j .
Suppose P_i^j is the number of party votes for party i in district j .

Suppose S_i is the number of seats that have been allocated to party i as described in section 3.2.

s_i^j is the number of seats already allocated to the district organization of party i in district j .

Stage 1:

For each j , we start with $s_i^j := D_i^j$.

Stage 2:

Repeat (until $\sum_j s_i^j = S_i$):

$t_i^j := P_i^j / (s_i^j + 0.5)$ for each district organization j of party i with $s_i^j > 0$.

$t_i^j := P_i^j / 0.75$ for each district organization j of party i with $s_i^j = 0$.

$T := \max_j t_i^j$.

$U := \max_j \{ D_i^j \mid t_i^j = T \}$.

The next seat goes to district organization j with $t_i^j = T$ and $D_i^j = U$. If there is more than one district organization with $t_i^j = T$ and $D_i^j = U$, then we decide randomly which district organization with $t_i^j = T$ and $D_i^j = U$ gets the next seat.

3.4. The Party Vote Winners

If $S_i^j > D_i^j$, then (in addition to those candidates who have already been elected by the district votes) those $S_i^j - D_i^j$ candidates of this party are elected who are ranked highest in the proportional ranking of this district.

Example: $D_{SPD}^{FK} = 2$ and $S_{SPD}^{FK} = 3$; then one (= $S_{SPD}^{FK} - D_{SPD}^{FK}$) additional SPD candidate must be elected in the district Friedrichshain-Kreuzberg; this additional candidate is Eggert, because he is the highest ranked SPD candidate in the proportional ranking of this district who has not yet been elected. $D_{Left}^{FK} = 2$ and $S_{Left}^{FK} = 3$; then one (= $S_{Left}^{FK} - D_{Left}^{FK}$) additional Left candidate must be elected in the district Friedrichshain-Kreuzberg; this additional candidate is Wolf, because he is the highest ranked Left candidate in the proportional ranking of this district who has not yet been elected. $D_{B'90G}^{FK} = 3$ and $S_{B'90G}^{FK} = 5$; then two (= $S_{B'90G}^{FK} - D_{B'90G}^{FK}$) additional B'90G candidates must be elected in the district Friedrichshain-Kreuzberg; these additional candidates are Behrendt and Altug, because they are the highest ranked B'90G candidates in the proportional ranking of this district who have not yet been elected.

Therefore, we get:

- | | |
|-------------------------------------------|---------------------------|
| 1. Mutlu (B'90G, candidate 04.002) | → elected (district vote) |
| 2. Fischer (SPD, candidate 01.006) | → elected (district vote) |
| 3. Reinauer (Left, candidate 03.009) | → elected (district vote) |
| 4. Junge-Reyer (SPD, candidate 01.001) | → elected (district vote) |
| 5. Ratzmann (B'90G, candidate 04.001) | → elected (district vote) |
| 6. İzgin (Left, candidate 03.005) | → elected (district vote) |
| 7. Klotz (B'90G, candidate 04.003) | → elected (district vote) |
| 8. Samuray (CDU, candidate 02.004) | → elected (district vote) |
| 9. Eggert (SPD, candidate 01.004) | → elected (party vote) |
| 10. Heinemann (SPD, candidate 01.007) | |
| 11. Behrendt (B'90G, candidate 04.011) | → elected (party vote) |
| 12. Wolf (Left, candidate 03.002) | → elected (party vote) |
| 13. Altug (B'90G, candidate 04.008) | → elected (party vote) |
| 14. Bayram (SPD, candidate 01.005) | |
| 15. Kosche (B'90G, candidate 04.010) | |
| 16. Michels (Left, candidate 03.001) | |
| 17. Bleiler (CDU, candidate 02.002) | |
| 18. Herrmann (B'90G, candidate 04.005) | |
| 19. Zackenfels (SPD, candidate 01.002) | |
| 20. Vordenbäumen (Left, candidate 03.007) | |
| 21. etc. | |

If the list of candidates of this party is exhausted, then this seat stays vacant.

4. Vacant Seats

When a seat gets vacant, then this seat goes to that candidate of this party who is ranked highest in the proportional ranking of this district. If the list of candidates of this party is exhausted or if this seat was the seat of an independent candidate, then this seat stays vacant.

Example: The seat of Klotz gets vacant. Then this seat goes to Kosche.

Therefore we get:

1. Mutlu (B'90G, candidate 04.002) → elected (district vote)
2. Fischer (SPD, candidate 01.006) → elected (district vote)
3. Reinauer (Left, candidate 03.009) → elected (district vote)
4. Junge-Reyer (SPD, candidate 01.001) → elected (district vote)
5. Ratzmann (B'90G, candidate 04.001) → elected (district vote)
6. İzgin (Left, candidate 03.005) → elected (district vote)
- ~~7. Klotz (B'90G, candidate 04.003) → elected (district vote)~~
8. Samuray (CDU, candidate 02.004) → elected (district vote)
9. Eggert (SPD, candidate 01.004) → elected (party vote)
10. Heinemann (SPD, candidate 01.007)
11. Behrendt (B'90G, candidate 04.011) → elected (party vote)
12. Wolf (Left, candidate 03.002) → elected (party vote)
13. Altug (B'90G, candidate 04.008) → elected (party vote)
14. Bayram (SPD, candidate 01.005)
15. Kosche (B'90G, candidate 04.010) → elected (successor of Klotz)
16. Michels (Left, candidate 03.001)
17. Bleiler (CDU, candidate 02.002)
18. Herrmann (B'90G, candidate 04.005)
19. Zackenfels (SPD, candidate 01.002)
20. Vordenbäumen (Left, candidate 03.007)
21. etc.